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T.Y.B.Sc. Mathematics (Sem. VI) Examination September - 2023

MTH - 602 - Linear Algebra - II

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(૧) **નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.**
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:
☛ **T.Y.B.Sc. Mathematics (Sem. VI)**

Name of the Subject :
☛ **MTH - 602 - Linear Algebra - II**

Subject Code No.: **2103000206020032**

Seat No.:

Student's Signature

(2) All questions are compulsory.
(3) Figures to the right indicate marks of the questions.
(4) Follow usual notations.

Q. 1 Answer the following questions. (Any Five) (10)

(1) Define : Zero map in a vector space. Prove that it is linear.

(2) Let T : $V_3 \to V_2$ be a linear map define by $T(x_1, x_2, x_3) = (x_1, x_1 + x_2)$. Is T a 1-1 linear map? Justify your answer.

(3) Find the linear map associate with a matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ relative to both standard basis.

(4) Let T : $V_3 \to V_2$ define by $T(e_1) = (2,1)$, $T(e_2) = (0,1)$, $T(e_3) = (2,0)$. Find $r(T)$.

(5) Find the range and rank of the matrix $\begin{bmatrix} 1 & 2 & -1 \\ -1 & 3 & 1 \end{bmatrix}$.

(6) Prove that A linear transformation T : $V_3 \to V_3$ is one-one then $n(t) = 0$.

(7) In an Inner product space V, prove that $||u + v|| \leq ||u|| + ||v||$, $\forall\ u, v \in V$.

(8) State difference between Orthogonal and Orthonornal set in an Inner product space V.

Q. 2 Answer the following. (Any two) **(10)**

(1) Let $T : U \to V$ be a linear transformation. Prove that
If $[u_1, u_2, u_3, ..., u_n] = U$ then $[T(u_1), T(u_2), T(u_3), ..., T(u_n)] = R(T)$.

(2) If $v_1, v_2, v_3, ..., v_n$ are n linearly independent vectors of $R(T)$ and if $u_1, u_2, u_3, ..., u_n$ are n vectors of U with condition that $T(u_i) = v_i$, $\forall\ i = 1$ to n for a linear map $T: U \to V$ then prove that $u_1, u_2, u_3, ... , u_n$ are $L.I$ vectors of U.

(3) Obtain the general rule of the linear transformation $T : V_3 \to V_3$ defined by $T(0,1,2) = (3,1,2)$, $T(1,1,1) = (2,2,2)$, $T(0,1,3) = (3,1,2)$.

Q. 3 Answer the following. (Any two) **(10)**

(1) Prove that the linear transformation $T : V_3 \to V_3$ such that $T(e_1) = e_1 - e_2$, $T(e_2) = 2e_2 + e_3$ and $T(e_3) = e_1 + e_2 + e_3$ is neither one- one nor on-to.

(2) Let $S : V \to W$ and $T : U \to V$ be two linear map then prove that

(a) ST is one-one then T is one-one

(b) ST is on-to then S is on-to.

(3) Let U be a vector space of dimension 4 then prove that $U \approx V_4$.

Q. 4 **Answer the following. (Any two)** **(10)**

(1) Find the matrix $(T; B_1, B_2)$ associated with a linear transformation $T : V_3 \to V_2$ defined by $T(X, Y, Z) = (X + Y, Y + Z)$ relative to basis $B_1 = \{(1,0,0), (1,1,0), (1,1,1)\}$ and $B_2 = \{(1,3), (\frac{1}{2}, 1)\}$.

(2) Find Range, Rank, Kernel and Nullity of the matrix $\begin{bmatrix} -1 & 1 & 1 \\ 3 & 1 & -1 \\ 2 & 1 & 1 \end{bmatrix}$

(3) Find the Linear transformation T associated with a matrix $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$ relative to basis $B_1 = \{(1,1,1), (1,2,3), (1,2,0)\}$ and $B_2 = \{(1,1), (1, -1)\}$.

Q. 5 **Answer the following. (Any two)** **(10)**

(1) In an Inner product space V, prove that $|u.v| \leq ||u||.||v||$, $\forall\ u, v \in V$.

(2) (a) In an Inner Product space V, Prove that $||\alpha\ v|| = |\alpha|\,||v||$, $\forall\ v \in V$ and $\propto$ a scalar.

(b) Is the statement " The norm of every vectors is a distance of it from the origin of the space" true ? in an Inner Product space V_n. Justify your answer.

(3) Orthonormalized the L.I set $\{(1,1,1), (1,0,0), (0,1,0)\}$ by Gram Schmidt's process.

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